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摘要

在本計劃補助下，我們完成了有關帶電矢量粒子在超強磁場下之粒子對產生的過渡效應問題。成果已發表在

Physical Review D, Vol. 60, pg. 033003 (1999)

題目為：Pair Production of charged vector bosons in supercritical magnetic fields at finite temperatures.

關鍵詞：粒子對產生率，超強磁場，矢量波色子

ABSTRACT

Under the support of this Grant, we have completed our studies of the effects of temperatures on the pair production of charge vector bosons in supercritical magnetic fields. Our result is published in

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Key words: Pair production rate, strong magnetic fields,
vector bosons.

The thermodynamic properties of an ideal gas of charged vector bosons (with mass m and charge e) is studied in a strong external homogeneous magnetic field no greater than the critical value $B_c = m^2/e$. The thermodynamic potential, after appropriate analytic continuation, is then used in the study of the spontaneous production of charged spin-one boson pairs from vacuum in the presence of a supercritical homogeneous magnetic field at finite temperature. [S0556-2821(99)02513-8]

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1. INTRODUCTION

As is well known [1] the energy spectrum of the vector boson with mass m , charge e , spin $S=1$, and gyromagnetic ratio $g=2$ in a constant uniform magnetic field $B=(0,0,B)$ is given by the formula (we set $\hbar=c=1$)

$$E_n(p) = \sqrt{m^2 + (2n+1-2S)\hbar e B} + p^2, \quad S = -1, 0, 1. \quad (1)$$

The integer n ($n=0,1,2,\dots$) labels the Landau level, and p is the momentum along the direction of the field. For $n=0$, $p=0$, $S=1$, E_0 vanishes at $B=B_c=m^2/e$. When $B>B_c$, E_0 becomes purely imaginary. Such behavior of the energy E reflects a quantum instability of an electrically charged vector boson field in the presence of an external uniform magnetic field. The source of this instability is due to the interaction of the external field with the additional (anomalous) magnetic moment of the bosons, which, owing to the gyromagnetic ratio $g=2$, appears already in the tree approximation.

Charged spin-1 particles with the gyromagnetic ratio $g=2$ are not minimally coupled to an external electromagnetic field (if they were coupled in such a way the above ratio had to be $g=1$). However, the quantum theory of relativistic charged spin-1 bosons with $g=2$ in the presence of external electromagnetic fields is a linear approximation of gauge field theory in which the local $SU(2)$ symmetry is spontaneously broken into $U(1)$ symmetry [2]. So one anticipates that the perturbative vacuum of the Weinberg-Salam electroweak model in the linear approximation would exhibit instability in a homogeneous superstrong magnetic field.

When B becomes equal to B_c the lowest energy levels of charged spin-1 particle and antiparticle "collide" with each other at $E_0=0$. One finds similar behavior in the case of scalar particles in a deep potential well which acts as the external field [3]. In this latter case, one usually interprets the behavior of energies as follows: when the binding energy of a state exceeds the threshold for particle creation, pairs of scalar particle-antiparticle may be spontaneously produced giving rise to the so-called condensate. The number of boson pairs produced by such a supercritical external field (here the

depth of the well) may be limited if only the mutual interaction of the created particles is taken into account. In the framework of (second) quantized field theory the behavior of bosons in supercritical external fields was first considered in Ref. [4], which assumes a self-interaction of the ϕ^4 -type, with the conclusion that the vacuum is, in fact, stabilized by the extremely strong (mutual) vacuum polarization. For a thorough discussion on the problem of electron-positron and scalar boson pair production in external electromagnetic fields see Ref. [5].

The case of vector bosons was considered in Ref. [6] by taking into account only the ground state of the charged spin-1 bosons in the superstrong external magnetic field and assuming a self-interaction for this state similar to the one of the W^\pm vector boson field in the Weinberg-Salam electroweak gauge theory, namely, the $|W|^4$ interaction. In this work the condensate energy of charged spin-1 boson pairs was found, and a scheme for quantizing the W field in the neighborhood of the new classical vacuum with $W_{\text{classical}} \neq 0$ near the threshold for condensate production $B=B_c \ll B_c = m_W^2/e$ (m_W is the mass of the W boson) was presented.

Using the complete electroweak Lagrangian the authors of Ref. [7] have managed to construct new "classical" static magnetic solutions for a W condensate in the tree approximation. They also show that the instability of the W field does not occur owing to the $|W|^4$ self-interaction term in the electroweak Lagrangian. Moreover, the electroweak gauge symmetry may be restored in the presence of a superstrong magnetic field with $B=m_W^2/e$ (m_W is the mass of the Higgs boson) if $m_H > m_W$.

In the one-loop approximation of the effective Lagrangian of the charged spin-1 boson field (without a self-interaction term), radiative corrections may induce, in the presence of a strong uniform magnetic field, the production of charged vector boson pairs in the lowest energy states, i.e., the condensate. It is of interest to see what happens with the vacuum when not only an external magnetic field is present but also when the temperature is finite.

In this paper we shall investigate the problem of pair production of charged vector bosons induced by the unstable mode in the presence of a supercritical magnetic field at fi-

may be spontaneously produced by a constant magnetic field when $B > B_c$ from the vacuum, just as electron-positron pairs are produced by an external electric field [13].

Before we proceed to the second mechanism, let us first give some estimates of the density of spin-1 boson pairs in the lowest energy state only that may be produced as a result of thermal collisions of real bosons in the external field with $B = B_c$. If the density of the created pairs is much greater than that of the bosons present initially, we may apply formula (11) with $\mu = 0$ to find the density of the pairs produced by thermal collisions. For low $(\beta M_- > 1)$ but $T \ll m$ and high $(\beta M_- < 1, T < m)$ temperature, we obtain, respectively,

$$\rho_T \approx \frac{eB(M_- T)^{1/2}}{(2\pi)^{3/2}} \exp(-M_-/T), \quad (16)$$

$$\rho_T \approx \frac{eBT}{2\pi^2} \ln(T/M_-). \quad (17)$$

Now we come to the second mechanism. As is known (see, for example, Ref. [14]) the quantum electrodynamics vacuum in the presence of an external electromagnetic field can be described by the total transition amplitude from the vacuum state $|0_{\text{vac}}\rangle$ in time $t \rightarrow -\infty$ to the vacuum state $\langle 0_{\text{vac}}|$ in time $t \rightarrow \infty$ as follows:

$$C_V = \langle 0_{\text{vac}} | 0_{\text{vac}} \rangle = \exp(iW(\mathcal{E}, B)), \quad (18)$$

where W is the effective action for a given quantum field. W defines the effective Lagrangian \mathcal{L}_{eff} according to $W = \int d^4x \mathcal{L}_{\text{eff}}$. The effective action is a classical functional depending on the external electric (\mathcal{E}) and magnetic (B) fields. When the external electromagnetic field is homogeneous the effective action is equal to $W(\mathcal{E}, B) = -[E(\mathcal{E}, B) - E(\mathcal{E} = 0, B = 0)]V\Delta t$, where $E(\mathcal{E}, B)$ is nothing but the density of vacuum energy in the presence of the external field. V is the volume, and Δt is the transition time. It is worthwhile to note that the effective action contains all divergencies of the theory but they are in the real part of $W(\mathcal{E}, B)$. C_V is the probability amplitude when the external electromagnetic field does not change and so this applies for the vacuum.

For external fields smoothly changing both in space and time one has

$$|C_V|^2 = \exp(2 \operatorname{Im} L_{\text{eff}}(\mathcal{E}, B)V\Delta t). \quad (19)$$

The imaginary part of the effective Lagrangian density $\operatorname{Im} L_{\text{eff}}$, or of the vacuum energy, is finite and describes production of particles by the external electromagnetic field. It also signals that an instability of the vacuum occurs. The imaginary part of the effective Lagrangian density reduces at $T=0$ to the imaginary part of the effective potential density. The latter (for the case under consideration) arises from the lowest energy of the charged massive vector boson being imaginary at $B > B_c$. At $T=0$ the imaginary part of the effective potential is [1,15]

$$\Omega(\mu) = \Omega_1(\mu) = -\frac{1}{2\pi^2\beta} \sum_{k=1}^{\infty} k^{-1} \exp(k\beta\mu) K_1(k\beta M_-). \quad (10)$$

Subsequently, the boson density is

$$\rho_B = \frac{eB M_-}{2\pi^2} \sum_{k=1}^{\infty} K_1(k\beta M_-) \exp(k\beta\mu). \quad (11)$$

When $M_- \gg T$, $M_- \mu < T$, we can get for the total density at equilibrium

$$\rho \approx \frac{eB(T M_-)^{1/2}}{(2\pi)^{3/2}} \left\{ \frac{\pi T}{M_- - \mu} \right\}^{1/2} - 1.46 \}. \quad (12)$$

The first (leading) term of Eq. (12) reduces exactly to the one obtained in Ref. [11]. The total boson density (11) for relatively "high" temperature for which $T \gg M_-$ but $T \ll m$ is

$$\rho \approx \frac{eBT}{2\pi^2} \exp(\mu T), \quad (13)$$

for $-\mu \gg T$ and

$$\rho \approx \frac{eBT}{2\pi^2} \ln(T/(M_- + \mu)) \quad (14)$$

for $-\mu \rightarrow M_-$.

It follows from formulas (12) and (14) that significant amount of vector bosons persists to fill the states with non-zero momentum projections on the magnetic field direction. These states now should be considered as excited ones. Hence, any density of bosons can be accommodated outside the ground state (with $\rho = 0$) at any temperature. This means that there is no true Bose-Einstein condensation in the presence of finite magnetic fields. We mention here that it was Feynman [12] who first showed that true BEC is impossible in a classical one-dimensional gas.

An exact expression for the magnetization of the vector boson gas in the lowest energy state in a strong magnetic field may be derived from Eq. (10) in the form [9]

$$M_z(B) = \frac{e}{2\pi^2\beta} \sum_{k=1}^{\infty} \left\{ \frac{M_-}{k} K_1(k\beta M_-) + \frac{eB}{2} K_0(k\beta M_-) \right\} \exp(k\beta\mu). \quad (15)$$

The magnetization is also a positive function of the magnetic field and temperature.

III. PAIRS PRODUCTION OF VECTOR BOSONS

Let us now turn to discussing the problem of pair production of vector bosons in a supercritical magnetic field at finite temperature. There are two possible mechanisms for this process: (1) pairs may be produced as a result of thermal collisions of real charged bosons in the external field, (2) pairs

may be produced by the thermal energy, one can approximate Ω as follows. For Ω_1 and Ω_2 in Eq. (3), we set $M_- \approx m(1 \mp \chi/2)$ with $\chi \approx eB/m^2$, and use the following formula

$$K_1(k\beta m(1 \mp \chi/2)) = K_1(k\beta m) + \chi \frac{dK_1(z)}{dz}, \quad (4)$$

where $z = k\beta m(1 \mp \chi/2)$. Evaluation of Ω_2 can be done by first replacing the summation over n by an integral using the Euler formula

$$\sum_{n=0}^{\infty} f(n+1/2) = \int_0^{\infty} f(x) dx + (1/24)f'(0), \quad (5)$$

and then by using the formula [10]

$$\int_0^{\infty} dz z^2 K_1(kmz\beta) = \frac{1}{km\beta} K_3(km\beta). \quad (6)$$

The thermodynamic potential and density of spin-1 bosons at equilibrium can then be obtained as

$$\Omega = -\frac{VT^{1/2}m^{3/2}}{(2\pi)^{3/2}} \left\{ 3T^{1/2} \operatorname{Li}_{3/2}(e^{\beta(\mu-m)}) + \frac{7(eB)^2}{8m^4} e^{\beta(\mu-m)} \right\}, \quad (7)$$

$$\rho = 3 \left\{ \frac{Tm}{2\pi} \right\}^{3/2} \zeta(3/2), \quad (8)$$

where $\operatorname{Li}_s(x) = \sum_{n=1}^{\infty} x^n/n^s$ is the polylogarithmic function of order s , and $\operatorname{Li}_s(1) = \zeta(s)$. The magnetization of the gas under the above conditions is a positive function of the magnetic field induction and temperature because paramagnetic (spin) contribution dominate

$$M_z(B) = -\frac{1}{V} \frac{\partial \Omega}{\partial B} = \frac{7e^2 B T^{1/2}}{4(2\pi)^{3/2} m^{1/2}} e^{\beta(\mu-m)}. \quad (9)$$

When $B \approx B_c$ transitions of bosons from level $n=0$ to any excited levels $n \neq 1$ will not be allowed if $T < eB/m$ and all bosons in quantum state with $n=0$ that are available may be considered as condensate in a two-dimensional "momentum" space in the plane perpendicular to the magnetic field with values of "effective momenta" $k < (eB)^{1/2}$. True condensate, however, will not actually be formed in three-dimensional momentum space because longitudinal momenta of bosons may have values outside this region.

For low temperature T such that $\beta M_- \gg 1$, contributions to the thermodynamic potential (3) from all the excited states of the vector bosons are exponentially small compared with that from the state with $n=0$ and $S=1$. Hence only the first term Ω_1 in Eq. (3) needs be considered in this limit

¹We take this opportunity to correct the expression for the magnetization in Ref. [9].

quantum statistical physics in the framework of standard quantum statistical quantities such as the thermodynamic potential are unambiguous and may be well defined. After deriving the thermodynamic potential in the region $B < B_c$, we shall perform an analytic continuation of this quantity into the supercritical region $B > B_c$. This will give us the imaginary part of the effective potential, from which we can derive the expression for the rate of pair production. The contribution in the thermal one-loop effective action from gauge boson field in a constant homogeneous magnetic field was previously considered in Ref. [8] in connection with the question of symmetry restoration. But in that work contribution from the unstable modes was explicitly ignored.

II. THERMODYNAMIC POTENTIAL

The thermodynamic potential Ω for a gas of real (not virtual) charged vector bosons as a function of the chemical potential μ , the magnetic induction of external field B , and the gas temperature $T \approx 1/\beta$ is defined by

$$\Omega = \frac{eBV}{4\pi^2\beta} \int d\rho \ln \{ 1 - \exp \beta [\mu - (m^2 - eB + \rho^2)^{1/2}] \} + \frac{eBV}{4\pi^2\beta} \sum_{n=0}^{\infty} \sum_{s_n} \int d\rho \times \ln \{ 1 - \exp \beta [\mu - (m^2 + (2n+1)eB + \rho^2)^{1/2}] \}, \quad (2)$$

where V is the volume of the gas, and $s_n = 3 - \delta_{n0}$ counts the degeneracy of the excited states.

By expanding the logarithms and integrating over ρ , one can recast Ω into [9]

$$\Omega(\mu) = \Omega_1 - \Omega_2 + \Omega_3, \\ \Omega_1 = -\frac{V e B}{2\pi^2\beta} \sum_{k=1}^{\infty} k^{-1} \exp(k\beta\mu) [M_-(k\beta M_-) - M_+(k\beta M_-)] - \frac{3VeB}{2\pi^2\beta} \sum_{n=0}^{\infty} \sum_{s_n} \sqrt{M_+^2 + 2neB} \\ \times \sum_{k=1}^{\infty} k^{-1} \exp(k\beta\mu) K_1(k\beta \sqrt{M_+^2 + 2neB}), \quad (3)$$

where $M_{\pm} = m^2 \mp eB$ and $K_n(x)$ is the modified Bessel function of order n . If both particles and antiparticles are present, the factor $\exp(k\beta\mu)$ in Eq. (3) has to be replaced by $\cosh(k\beta\mu)$. The thermodynamic potential as a function of chemical potential is real-valued for real values of μ that satisfy $|\mu| \leq M_-$. This condition comes from the physical requirement that the density (and the occupation numbers) of particles and antiparticles with the mass M_- is positive for any real values of momenta p .

